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A COMPOSITE, MULTILAYERED CYLINDRICAL DIELECTRIC RESONATOR

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Abstract

The analysis of TE modes in a composite multilayered cylindrical dielectric resonator is presented. All cylindrical dielectric resonator structures which have found some practical applications can be treated as a special case of the structure under consideration. These include: single resonator, double resonator, ring resonator, resonator in microstrip, inhomogeneous resonator and others. A new rigorous method of determining the field distribution and resonant frequency of this resonator is presented. By this method the solution is obtained in a form of successive approximations converging to an exact solution. The analysis is applied in detail to the lowest frequency TE mode of double dielectric resonator.

Introduction

In the past several years dielectric resonators have found numerous applications in miniature microwave components and subsystems [1]. Many different dielectric resonator structures usually possessing circular symmetry and excited in TE mode were employed. They were: dielectric sample between parallel conducting plates [2], [3], dielectric sample on microstrip substrate [2], [4], [5], inhomogeneous dielectric resonator [6], double dielectric resonator [7], [8], ring dielectric resonator [8], to mention a few. All of these structures can be treated as a special case of a structure shown schematically in Figure 1, called a composite multilayered cylindrical dielectric resonator. The method of analysis of TE modes in this structure and an example of application to the lowest TE mode of a double dielectric resonator is presented.

Method of Analysis

For TE-modes fundamental equation to be solved is:

$$\frac{\partial^2 E_\phi}{\partial z^2} + \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial (rE_\phi)}{\partial r}\right] + k_o^2\, \varepsilon_r(r,z)\, E_\phi = 0 \quad (1)$$

which is obtained in elementary way from Maxwell's equations. Equation (1) is solved separately in regions I, II, III (Figure 1) by the method of separation of variables, as ε_r is only a function of z within each region.

As $$E_\phi^M(r,z) = \Phi^M(z)U^M(r) \qquad M = I,\ II,\ III \quad (2)$$

equation (1) becomes:

$$\frac{d}{dr}\left[\frac{1}{r}\frac{d(rU^M(r))}{dr}\right] + t_m^M U^M(r) = 0 \quad (3)$$

$$\frac{d^2\Phi^M(z)}{dz^2} + (k_o^2\varepsilon_r^M(z) - t_m^M)\,\Phi^M(z) = 0 \quad (4)$$

Let $d_i^M = \sum_{j=1}^{i} h_j^M$ for $i = 1, 2 \ldots \ell^M$, then in every

interval $[d_{i-1}, d_i]$: $$(v_{mi}^M)^2 = k_o^2\varepsilon_i^M - t_m^M \quad (5)$$

and the functions $\Phi_m^M(z)$, orthogonal in the interval $[0,L']$, satisfying (4) and boundary conditions can be constructed as follows:

-in interval $[0,d_1^M]$: $$\Phi_m^M(z) = \frac{1}{v_{m1}} \sin(v_{m1}z) \quad (6)$$

-in interval $[d_{i-1}^M, d_i^M]$:

$$\Phi_m^M = \frac{\Phi_m'^M(d_i^M)}{v_{mi}^M}\sin(v_{mi}^M(z-d_i^M)) + \Phi_m(d_i^M)\cos(v_{mi}^M(z-d_i^M))$$

where $$\Phi'^M_m(d_i^M) = \left.\frac{d\Phi_m^M(z)}{dz}\right|_{z=(d_i^M)^-} \quad (7)$$

For $z = L'$, t_m^M of equation (5) has to be chosen to satisfy:

$$\Phi_m^M(L') = \Phi_m^M(d_{\ell^M}^M) = 0$$

The solution of (3) is given by linear combinations of appropriate Bessell functions $J_1(h_m^M r)$ and $N_1(h_m^M r)$ or $I_1(h_m^M r)$ and $K_1(h_m^M r)$ where $h_m^M = \sqrt{|t_m^M|}$.

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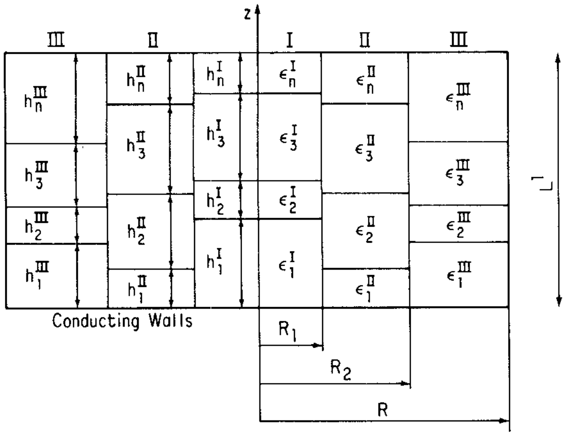

Fig. 1. A composite multilayered cylindrical dielectric resonator structure.

Therefore E_ϕ is given by:

in region I: $$E_\phi^{I}(r,z) = \sum_{m=1}^{\infty} a_m^{I} \,\Phi_m^{I}(z)\, J_1(h_m^{I} r) \quad (8a)$$

in region II: $$E_\phi^{II}(r,z) = \sum_{m=1}^{\infty} [a_m^{II} J_1(h_m^{II} r) + b_m^{II} N_1(h_m^{II} r)]\Phi_m^{II}(z) \quad (8b)$$

in region III:

$$E_\phi^{III}(r,z) = \sum_{m=1}^{\infty} a_m^{III} [N_1(h_m^{III} r)\, J_1(h_m^{III} R) - J_1(h_m^{III} r)\, N_1(h_m^{III} R)]\, \Phi_m^{III}(z) \quad (8c)$$

The application of boundary conditions at $r = R_1$ and $r = R_2$ results in the infinite set of homogeneous linear equations, which possesses nontrivial solution if the determinant of the coefficient matrix $\underset{\sim}{W}$ vanishes. Namely

$$\det\{\underset{\sim}{W}\} = \det\{(\underset{\sim}{W}_A^{I})^{-1} \underset{\sim}{W}_B^{I} - (\underset{\sim}{W}_A^{III})^{-1} W_B^{III}\} = 0 \quad (9)$$

The elements of matrices $\underset{\sim}{W}_A^{I}$, $\underset{\sim}{W}_B^{I}$, $\underset{\sim}{W}_A^{III}$, $\underset{\sim}{W}_B^{III}$ are given by:

-matrix $\underset{\sim}{W}_A^{I}$: $$W_{im} = \langle\phi_m^{II},\phi_i^{I}\rangle \left[\frac{J_1(h_m^{II}R_1)}{J_1(h_i^{I}R_1)} - \frac{h_m^{II}J_o(h_m^{II}R_1)}{h_i^{I}J_o(h_i^{I}R_1)}\right]$$

-matrix $\underset{\sim}{W}_B^{I}$: $$W_{im} = \langle\phi_m^{II},\phi_i^{I}\rangle \left[\frac{h_m^{II}N_o(h_m^{II}R_1)}{h_i^{I}J_o(h_i^{I}R_1)} - \frac{N_1(h_m^{II}R_1)}{J_1(h_i^{I}R_1)}\right]$$

-matrix W_A^{III}:

$$W_{im} = \langle\phi_m^{II},\phi_i^{III}\rangle \left[\frac{J_1(h_m^{II}R_2)}{N_1(h_i^{III}R_2)J_1(h_i^{III}R) - J_1(h_i^{III}R_2)N_1(h_i^{III}R)} - \frac{h_m^{II}J_o(h_m^{II}R_2)}{h_i^{III}[N_o(h_i^{III}R_2)J_1(h_i^{III}R) - J_o(h_i^{III}R_2)N_1(h_i^{III}R)]}\right]$$

matrix W_B^{III}: (10)

$$W_{im} = \langle\phi_m^{II},\phi_i^{III}\rangle \left[\frac{h_m^{II}N_o(h_m^{II}R_2)}{h_i^{III}[N_o(h_i^{III}R_2)J_1(h_i^{III}R) - J_o(h_i^{III}R_2)N_1(h_i^{III}R)]} - \frac{N_1(h_m^{II}R_2)}{N_1(h_i^{III}R_2)J_1(h_i^{III}R) - J_1(h_i^{III}R_2)N_1(h_i^{III}R)}\right]$$

where $\langle \Phi,\Psi \rangle = \int_0^{L'} \Phi(z)\,\Psi(z)\,dz$

In the eqs. (9) and (10) for $t_j^M < 0$ the functions J_o, N_o, J_1, N_1 should be replaced by I_o, K_o, I_1, K_1, respectively. Eq. (9) can be solved to find the resonant frequency to any desired accuracy by appropriate choice of matrix $\underset{\sim}{W}$ dimension. Then coefficients a_m^M and b_m^M can be found to determine the field distribution inside the resonator.

Double Dielectric Resonator Problem

The double dielectric resonator structure is shown in Figure 2. The samples 2 and 4 are made of high-ε dielectric material and samples 1, 3, 5 are made of low-ε dielectric material. This structure can be considered as a special case of the structure in Figure 1 if $R_1 = R_2$ and $R \to \infty$.

A comparison between the resonant frequency of lowest TE-mode computed by this method, three different approximate methods, and experimental data are shown in Figure 3 and Table I.

The solution for the dimension of the matrix [W] N = 10 and experimental data agree within the estimated accuracy of the experiment. The solution for N = 1, which results in very simple, calculator programmable model, provides much better accuracy than any of the other approximate methods of analysis, that is: a) magnetic wall waveguide model [7]; b) dielectric waveguide model [10]; c) modified dielectric waveguide model [2]. The accurate method of analysis presented for single dielectric resonator in [3] runs into computational problems for double resonator case. The comparison of these two accurate methods in the case of a single dielectric resonator shows faster convergence of the method outlined in this paper.

Conclusions

The method of analysis outlined in this paper allows investigation of much more complex resonant structures than any of the rigorous methods previously reported (for instance [3, 9, 11]). This method should prove to be very useful in: 1) design of dielectric resonators with pre-determined temperature dependence of the resonant frequency; 2) investigation of tuning methods; 3) design of supporting structures; 4) measurement of microwave properties of dielectrics.

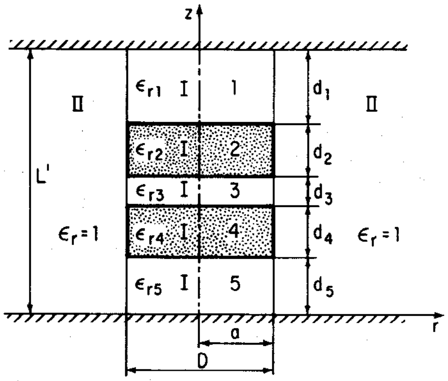

Fig. 2. A double cylindrical dielectric resonator structure.

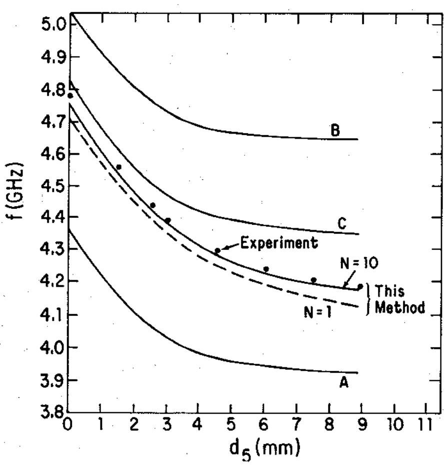

Fig. 3. Comparison between the resonant frequency of lowest TE-mode for the structure in Fig. 2 computed by the new method for N=1 and N=10, three different approximate methods and experimental data.
d_1=1.92 mm, d_2=d_4=3.52 mm, d_3=3.48 mm, D=14.98 mm, $\varepsilon_{r1}=\varepsilon_{r3}=\varepsilon_{r5}=2.04$, $\varepsilon_{r2}=\varepsilon_{r4}=34.1$.
A - magnetic wall waveguide model [7]; B - dielectric waveguide model [10]; C - modified dielectric waveguide model [2].

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				APPROXIMATE METHODS						THIS METHOD			
DIMENSIONS [mm]			EXPER.	A		B		C		N=1		N=10	
d_1	d_3	d_5	f_o MHz	f_o MHz	Error %	f_o MHz	Error %	f_o MHz	Error %	f_o MHz	Error %	f_o MHz	Error %
1.92	3.48	8.93	4182	3926	6.1	4648	11.2	4357	4.2	4128	1.2	4180	0.05
1.92	3.48	2.48	4433	4071	7.9	4761	7.7	4512	2.1	4394	0.6	4420	0.30
2.48	1.40	4.48	4061	3716	8.1	4319	6.8	4138	2.3	4034	0.2	4043	0.45
2.48	1.40	0.0	4541	4126	9.0	4735	4.5	4576	0.9	4528	0.1	4533	0.18

Table I. Comparison between the resonant frequency of lowest TE-mode for the structure in Fig. 2 computed by the new method for N=1 and N=10, three different approximate methods and experimental data: A - magnetic wall waveguide model [7], B - dielectric waveguide model [10], C - modified dielectric waveguide model [2].
d_2=d_4=3.52 mm, D=14.98 mm, $\varepsilon_{r2}=\varepsilon_{r4}=34.1$, $\varepsilon_{r1}=\varepsilon_{r3}=\varepsilon_{r5}=2.04$